ANNOTATION
of the dissertation work by G.T. Korpebay
on the topic:
"*Integral Equations in the Theory of Controllability and Time-Optimal Problems for Ordinary and Linear Integro-Differential Equations*"

The dissertation work is devoted to the study of controllability and optimal fast performance problems of dynamic systems with phase, integral, and control constraints. The work considers linear, nonlinear, and linear integro-differential dynamic systems for which constructive algorithmic methods for solving the corresponding control problems have been developed. Particular attention is paid to the construction of computational algorithms, their software implementation, and the verification of the effectiveness of the proposed methods using model problems.

Relevance of the Research Topic

The modern theory of control of dynamic systems occupies an important place in mathematics, mechanics, and applied sciences. Optimal control methods are widely used in the study and control of various dynamic processes arising in physics, mechanics, energy systems, robotics, economics, and many other fields.

One of the central directions in control theory is the study of controllability and optimal control problems. An important role in this direction is played by optimal fast performance problems, in which it is required to transfer a system from one state to another in minimal time under various constraints on the system states and control actions.

In classical optimal control theory, a number of fundamental methods for studying such problems have been developed, including the Pontryagin Maximum Principle, Bellman's Dynamic Programming Method, and other approaches. These methods make it possible to establish important qualitative properties of optimal control problems, such as the existence of optimal control and necessary optimality conditions.

However, these methods are mostly qualitative in nature and do not always allow one to obtain algorithms for the practical computation of control actions. This is especially evident in problems with phase and integral constraints, as well as for nonlinear and integro-differential systems. In such problems, the development of constructive methods and solution algorithms represents a significant scientific and practical challenge.

In this regard, it is relevant to develop methods that allow not only establishing the theoretical properties of controllability and optimal fast performance problems but also constructing algorithms for computing controls suitable for numerical implementation and computer modeling.

Purpose and Objectives of the Research

The purpose of the dissertation work is to develop constructive methods for solving controllability and optimal fast performance problems of linear, nonlinear,

and linear integro-differential dynamic systems with phase and integral constraints, oriented toward algorithmic implementation and numerical modeling.

To achieve this goal, the following tasks are addressed in the dissertation:

– obtaining controllability conditions for linear dynamic systems with phase and integral constraints;

– developing algorithms for computing admissible controls and determining the optimal control time;

– constructing gradient methods for solving controllability problems for nonlinear dynamic systems;

– developing computational algorithms for linear integro-differential systems;

– implementing the proposed algorithms in software and conducting computational experiments on model problems.

Research Methods

The dissertation uses the method is based on the study of solvability and construction of solutions of the Fredholm integral equation of the first kind.

In addition, the following methods are used:

– methods of functional analysis;

– minimization methods in Banach spaces;

– numerical optimization methods;

– computational modeling methods.

Scientific Novelty of the Work

The scientific novelty of the dissertation consists of the following results:

Constructive algorithms for finding admissible controls for controllability problems of linear, nonlinear, and linear integro-differential dynamic systems with phase and integral constraints have been developed.

Algorithmic procedures for solving optimal fast performance problems for the specified classes of systems have been proposed, ensuring the constructive computation of optimal control.

A software implementation of the developed algorithms in the Python programming language has been created, and computational experiments on model controllability and optimal fast performance problems have been carried out, confirming the effectiveness and applicability of the proposed methods.

Main Research Results

A number of theoretical and applied results have been obtained in the dissertation.

For linear dynamic systems, algorithmically implementable methods for solving controllability and optimal fast performance problems under phase and integral constraints have been developed. Computational procedures for determining admissible controls and optimal control time have been constructed. The proposed methods are confirmed by computational experiments.

For nonlinear systems, the controllability problem is reduced to a functional minimization problem. Gradient algorithms for solving the corresponding optimization problems have been developed and their convergence has been investigated. Numerical simulations were performed using examples such as the Duffing equation and the mathematical pendulum, demonstrating the consistency between theoretical conditions and numerical results.

For linear integro-differential systems, algorithms for computing admissible controls and boundary states have been developed. The computational experiments performed confirmed the efficiency of the proposed methods and their suitability for practical applications.

Software Implementation

An important result of the work is the creation of a software package in Python, which provides numerical solutions to controllability and optimal fast performance problems.

The developed software allows:

– computing admissible controls for different classes of dynamic systems;

– determining optimal control time;

– modeling the dynamics of controlled systems;

– visualizing system trajectories and control actions.

The conducted computational experiments confirmed the correctness of the theoretical results and the effectiveness of the developed algorithms.

Theoretical and Practical Significance

The theoretical significance of the work lies in the development of the constructive direction of modern optimal control theory and the expansion of methods for studying controllability problems of dynamic systems.

The practical significance of the obtained results is determined by the possibility of applying the developed algorithms and software tools in modeling and controlling dynamic processes in various applied fields, including technical systems, robotics, and energy systems.

Reliability of the Results

The reliability of the obtained results is ensured by strict mathematical proofs, the use of modern methods of mathematical analysis, and confirmation by numerical experiments.

Approbation of the Results

The main results of the dissertation were presented and discussed at international scientific conferences and scientific seminars and were also published in scientific journals and conference proceedings.

Scientific Conclusions, Their Reliability and Validity

The results obtained during the research are supported by rigorous proofs previously presented by well-known scientists, publications in indexed international scientific journals, and materials from international scientific conferences.

Publications

Based on the results of the dissertation research, 10 scientific works have been published:

1. Aisagaliev, S., Korpebay G. Controllability and optimal speed-in-action of linear systems with boundary conditions // Bulletin of the Karaganda University. Mathematics Series. – 2023, Vol.110(2). – P. 21–34.

2. Aisagaliev, S., Korpebay G. Controllability and optimal fast operation of nonlinear systems // Bulletin of the Karaganda University. Mathematics Series. – 2024, Vol.115(3). – P. 77–92.

3. Korpebay G.T. Optimal fast performance of linear systems with phase and integral constraints // Ufa Autumn Mathematical School, Ufa, Russia, October 1, 2022, pp. 125–127.

4. Korpebay G.T. Controllability and optimal fast performance of linear ordinary differential equations with phase and integral constraints // International Scientific and Practical Conference “Digital Technologies and Interdisciplinary Approaches to Solving Applied Problems of Science”, H. Dosmukhamedov Atyrau University, Atyrau, October 6, 2025, pp. 154–155.

5. Korpebay G.T. Integral equations in the theory of controllability and optimal fast performance of ordinary and linear integro-differential equations // Current Issues of Fundamental and Applied Mathematics, November 18–21, 2025, Ust-Kamenogorsk.

6. Korpebay G. Controllability and optimal performance of linear systems with phase constraints // IX International Scientific and Practical Conference “Modern Science: Fundamental and Applied Aspects”, June 25–26, 2024, Rome, Italy, pp. 18–20.

7. Korpebay G. Boundary value problems of linear control systems with phase and integral constraints // XIV International Scientific and Practical Conference “The Modern Vector of the Development of Science”, June 27–28, 2024, Philadelphia, USA, pp. 41–43.

8. Korpebay G.T. Solvability of the Fredholm integral equation of the first kind and a method for constructing an approximate solution // International Scientific Conference of Students and Young Scientists “Farabi Alemi”, Almaty, Kazakhstan, April 6–7, 2023, p. 19.

9. Korpebay G.T. Controllability of linear systems with phase and integral constraints // V International Book Edition of the Countries of the Commonwealth of Independent States, Nur-Sultan, 2022, pp. 62–72.

10. Korpebay G.T. Controllability and optimal fast performance of linear systems // VIII International Book Edition of the Countries of the Commonwealth of Independent States, Nur-Sultan, 2023, pp. 42–44.

Significance of the Results and Prospects for Further Research

A distinctive feature of the conducted research is the transition from the theoretical analysis of controllability problems to the development of algorithms for

their practical solution. The proposed approach makes it possible to combine theoretical methods for studying dynamic systems with modern computational technologies.

The obtained results show that the developed algorithms can be effectively applied in solving control problems for various classes of dynamic systems. The implementation of algorithms in the Python programming language allows conducting computational experiments and studying the behavior of controlled systems under different parameters.

The proposed methods can serve as a basis for further research in the field of constructive methods of control theory. A promising direction is the development of these algorithms for more complex classes of dynamic systems, including distributed parameter systems, stochastic models, and systems with delays.

Thus, the conducted research contributes to the development of modern optimal control theory aimed at creating algorithmically implementable methods for solving control problems of dynamic systems.

Structure and Volume of the Dissertation

The dissertation consists of an introduction, three chapters, a conclusion, a list of references, and an appendix. The total volume of the dissertation is 175 pages.

The first chapter investigates linear systems with phase and integral constraints. The obtained theoretical results are developed into algorithms for computing admissible controls and optimal control time.

The second chapter considers nonlinear dynamic systems. The controllability problem is reduced to a functional minimization problem, gradient computational methods are constructed, and numerical solutions of model problems are carried out.

The third chapter studies linear integro-differential systems. Algorithms for computing controls are developed and computational experiments confirming the applicability of the proposed methods are performed.

The obtained theoretical results are implemented in the form of a Python software package that provides numerical solutions to controllability and optimal fast performance problems, modeling of system dynamics, and visualization of results.